M.A./M.Sc. (4th Semester)
Examination, June-2021
MATHEMATICS
Paper-VI
FLUID MECHANICS-II

Time : Three Hours]   [Maximum Marks : 80

[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Answer all the questions:-   1x10=10

   (a) For an irrotational motion what is the value of circulation.

   (b) The strength K of circular vortex is given by........

   (c) Kinetic energy of is rotational flow is given by $T =$?

   (d) What is stokes stream function? Explain.

   (e) Write down the solution of one dimensional wave equation.

   $$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}$$

   (f) When a circular cylinder in placed in a uniform stream, the couple acting on the cylinder is...........

   (g) Write down the equation of lines of flow relative to R sphere.

   (h) Write down the differential equations of vortex lines.

   (i) What are the vortex lines at an internal point in a fluid.

   (j) If ϕ be the velocity potential due to a simple three dimensional source, then write down the value of ϕ in used symbol.

2. Answer the following questions:-   2x5=10

   (a) Define vortex line.

   (b) Define conservation of velocity.

   (c) Define Karman street.

   (d) What are long gravity waves, (Explain).

   (e) To find the K.E. when are elliptic cylinder notates in an infinite man of liquid at rest at infinity.

Section-B   12x5=60

Answer the following questions:-

3. Show that with proper choice of units the motion of an infinite liquid produced by the motion of an elliptic cylinder parallel to one of its principal axes is given by the complex function $w = e^{-\xi}$, where $z = 2cosh\xi\ D$

   $$x = \phi\left(1 + \frac{1}{\phi^2+\psi^2}\right), \qquad y = \psi\left(1 - \frac{1}{\phi^2+\psi^2}\right)$$

   OR

   A thin shell in the form of an infinitely long elliptic cylinder, semi-axes a and b, is rotating about its axes in an infinite liquid otherwise at rest. It is filled with the same liquid prove that the ratio of the Kinetic energy of the liquid inside to that of the liquid outside is

   $$2ab: a^2 + b^2$$

4. An infinite ocean of an in compressible perfect liquid of density p is streaming past a fixed spherical obstacle of radius a. the velocity is uniform and equal to U except in so for as it is distributed by the sphere, and the pressure in the liquid at a great distance from the obstacle in π. Show that the thrust on that half of the sphere on which the liquid impinges in $\pi a^2 \left[\pi - \frac{1}{16} p U^2 \right]$

OR

Prove that at a point on the sphere moving through an infinite liquid the pressure is given by formula.

$$\frac{p - p_0}{\rho} = \frac{1}{2} a f \cos\theta_1 + \frac{1}{8} v^2 (9\cos^2\theta - 5)$$

Where v is the velocity, f the acceleration of the sphere, and θ, θ_1 are the angles between the radius and the devotions of v, f respectively, and in p_0 is the hydrostatic pressure.

5. Find the necessary and sufficient condition that vortex lines may be at night angles to the stream lines and if

$$U = \frac{ax - by}{x^2 + y^2}, \qquad V = \frac{ay + bx}{x^2 + y^2} \qquad and \quad w = 0$$

Investigate the nature of the motion of the nature of the motion of the liquid.

OR

An elliptic cylinder is filled with liquid which has mole-cular rotation w at every point, and whose particles move in planes perpendicular to the axes; prove that the stream lines are similar ellipses described in periodic time, $\frac{\pi}{w} \frac{a^2 + b^2}{ab}$

6. To find the path of a vortex in the angle between two planes to which it is parallel.

OR

Verify that the steam function for uniform streaming parallel to the axis port a solid bounded by those parts of the circles

$$(x + 1)^2 + y^2 = 2 ; \qquad (x - 1)^2 + y^2 = 2$$

7. Show that the group velocity for deep sea wave is half the wave velocity.

OR

Find the dispersion equation for place sound waves in air, accounting for viscosity and heat conduction.